

Sample Syllabi for DMI modules.

The syllabi below were written according to the specifications required at one university. You may choose to adapt them to meet the requirements of your context.

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BST Syllabus

Building a System of Tens: Calculation with Whole Numbers and Decimals

Course Description:

Building a System of Tens is designed to help experienced K-8 teachers explore the structure of our base-10 number system including whole numbers and decimals and examine how children develop an understanding of the number system. At the heart of the course are sets of classroom episodes (cases), illustrating student thinking as described by their teachers. Besides case discussions, the curriculum offers teachers opportunities to view and discuss videotapes of mathematics classrooms and interviews with mathematics students; to explore mathematics in lessons lead by the instructor; to share and discuss the work of their own students; to plan, conduct, and analyze mathematics interviews of students; to analyze lessons taken from elementary mathematics curricula; and to read and reflect on overviews of related research.

Learning Outcomes:

The goal of this course is to help teachers learn how to:

- define and select mathematical objectives for their students,
- recognize key mathematical ideas with which their students are grappling,
- support their students' mathematical thinking,
- appreciate the power and complexity of student thinking,
- ask questions that deepen students' mathematical understandings,
- analyze curriculum for the mathematics it contains,
- adapt curriculum to meet individual student's needs,
- make mathematical connections for themselves, and
- connect their experiences to research.

Course Requirements:

Class 1

Writing Assignment #1: Collect a work sample from three of your students involving multi-digit computation or, if your students are young, on understanding numbers between ten and twenty. Choose one whose work you think is strong and two whose work is not so strong. For each piece of student work answer these three questions: What does the student understand? What is the student missing? What is your learning goal for each student?

Reading Assignment #1: Read the introduction to Chapter 1 and the five cases in Building a System of Tens. Use the questions posed in the introduction to guide your reading.

Class 2

Writing Assignment #2: Consider these two methods that students in the video clips used to determine $123 - 76$:

I changed the 123 to 120 and the 76 to 70. Then I did $120 - 70 = 50$. Then I added the 3 to the 50, and then I took the 6 away and got 47.

I did the inverse operation. I did $76 + \text{something} = 123$. I tried to get to 100. That would be 24 and then there is 23 more. So it is 47.

Apply both approaches to a different subtraction problem. Represent each strategy with a base ten model or a number line. Write a verbal description of the strategy. In what way does each strategy take advantage of the base ten structure of numbers? What mathematical ideas are needed to understand each of these?

Reading Assignment #2: Read Chapter 2 in the casebook for Building a System of Tens, including the introduction and the six cases. As you read, consider the issues raised in the chapter introduction.

Class 3

Interview and Writing Assignment #3: Conduct a mathematical interview with a student in which you explore his or her ideas about the number system, drawing on what we have done in class so far. Write about your plans, what actually happened, what you learned, what surprised or confused you, and what questions the interview raised for you. Give enough information so the reader can understand the highlights of what happened and what you learned.

Reading Assignment #3: Read Chapter 3 in the casebook for Building a System of Tens, including the introduction and the three cases. As you read, consider the issues raised in the chapter introduction.

Class 4

Writing Assignment #4: Analyzing subtraction algorithms

Consider these two approaches for solving the subtraction problem $53 - 17$.

(a) $53 - 17 = 56 - 20 = 36$

(b)
$$\begin{array}{r} 4\cancel{5} \overset{1}{3} \\ - 1 \overset{7}{7} \\ \hline 3 \overset{6}{6} \end{array}$$

Do the following to analyze each approach:

- Create a representation for the strategy using the kind of models explored in the session.
- Try another subtraction problem using this procedure.
- Explain how procedure works; that is, why does it produce a correct answer?
- Make a list of the mathematical ideas involved in each procedure.

Reading Assignment #4: Read Chapter 4 in the casebook for Building a System of Tens, including the introduction and the four cases. As you read, consider the issues raised in the chapter introduction.

Class 5

Writing Assignment #5: Pose a mathematics task to your students related to computation of multidigit numbers. (If you work with students for whom this might not be appropriate, focus on the way they understand numbers between 10 and 20.) You might pose a question taken directly from one of the cases or from the math activities from the seminar, such as exploring a subtraction or multiplication problem using mental math, and representations such as number lines or base ten drawings. Write up your question, how your students responded, and what you make of their responses (your expectations, your surprises, and what you learned). Include specific examples of student work or dialogue. Reporting in detail about the work of a few students is very helpful. In particular, it is useful to analyze the work of students whose work might be confusing.

Reading Assignment #5: Read Chapter 5 in the casebook for Building a System of Tens, including the introduction and the four cases. As you read, consider the issues raised in the chapter introduction.

Class 6

Writing Assignment #6A: This assignment is about the math *you* are learning in the seminar, not about the math learning of your students. Reflect on the mathematics you have been thinking about in this seminar. Chose one topic to write about. Explain how you thought about this originally, what makes sense to you now, and what aspect of the idea are you still working on.

Writing Assignment #6A: Write answers to the questions on the “Examining Curriculum Activities” worksheet for the particular activity you are examining.

Reading Assignment #6: Read Chapter 6 in the casebook for Building a System of Tens, including the introduction and the five cases. As you read, consider the issues raised in the chapter introduction.

Class 7

Writing Assignment #7: Pose a mathematics task to your students related to the work of this seminar. Write up your question, how your students responded, and what you make of their responses (your expectations, your surprises, and what you learned). Include specific examples of student work or dialogue.

Reading Assignment #7: Read Chapter 7 in the casebook for Building a System of Tens, including the introduction and the three cases. As you read, take note of specific decisions or actions of the teachers.

Class 8

Writing Assignment #8: For this assignment, you will write your analysis of the moves a teacher makes (in a case from the casebook) and the impact of that move on the students. Analyze each passage by responding to these questions: What did the teacher do? What can you infer about the teacher's agenda for her students? How does the teacher's action connect with her assessment of their understanding? What do you think the teacher is trying to accomplish? What is the impact of the move on the students' thinking?

Reading Assignment #8: Read Chapter 8 in the casebook for Building a System of Tens, which takes another look at the mathematical issues in the cases through the essay "Highlights of Related Research." You might first reread the casebook "Introduction" (p. 1) for an overview of the broad themes explored in the cases. As you read, take notes on these three questions: What did you learn from the research article that wasn't in the cases? Identify two points in the essay that particularly interested you. Explain what made them interesting. What connections do you see between your classroom and the ideas in the essay?

Writing Assignment #9: Personal Reflections: Reflect and write about your experience in this course. Throughout the *Building a System of Tens* seminar, you have been compiling a portfolio that contains your reflective writing, examples of student thinking, and cases you have written, as well as facilitator's responses to your writings. Look through the material you have collected to see how your ideas about *learning*, about *mathematics*, or about *teaching* have changed. Write a reflective paper about your changing ideas in regard to one of these three topics, referencing specific items in your portfolio as illustrations.

Evaluation:

5% each	Writing Assignment 1 (total 5%)
10% each	Writing Assignments 2, 4, 6, 8, and 9 (total 50%)
15% each	Writing Assignments 3, 5 and 7 (total 45%)

Methodology:

Analysis of print and video cases happens in both small group and whole class discussions. Participants work with partners to explore the mathematics in both adult and student activities presented by the instructor. Their observations and questions become the focus for class discussions. Participants also present their ideas about their students' work and about their student mathematical interviews. The instructor guides the discussion to highlight important mathematical issues and also circulates during small group and partner work to raise questions that will push participants' thinking.

Topical Outline:

Class 0 (1 hour): Introduction

Overview of course and requirements

Introduction to working with print and video cases

Explanation of initial assignment

Class 1: Students' Addition and Subtraction Strategies

Sharing children's work samples, identifying strengths, weaknesses, and goals

Mental math activity: sharing addition and subtraction strategies

DVD: Subtraction listening to 2nd and 7th grade students explain their own strategies

Case discussion: identifying student strategies, considering what children need to understand to devise their own strategies

Math activity: Close to 100 and identifying strategies and their relationship to the number system

Class 2: Base-Ten Structure of Numbers

Discussion of learning environment: making a safe place for everyone's ideas

Math activity: Examining multiplying by ten

DVD: discussing students' mathematical understandings and examining the interview process

Case discussion: decomposing and recombining numbers and relating written and spoken numbers

Class 3: Making Sense of Addition and Subtraction Strategies

Math activity: exploring the ideas underling both invented and familiar computational strategies

DVD: Analyzing student's strategies for addition and subtraction

Case discussion: identifying students' addition and subtraction procedures

considering their teachers' reactions and considering the place of the traditional algorithms

Class 4: Multiplication of Multidigit Numbers

Math activity: modeling and solving multiplication problems

DVD: Multiplication

Discussion of cases and video: identifying students' multiplication strategies for decomposing and recombining numbers in multiplication

Class 5: Division of Multidigit Numbers

Discussion of homework assignment and sharing reflections on cases

Case discussion: decomposing and recombining numbers in division

DVD: Students' Approaches to Division: identifying students' division strategies and considering what students understand about division

Math activity: procedures for division

Class 6: Place Value Representation of Numbers Less than 1
Math activity: representing, adding, and subtracting decimals
Case discussion: reconsidering place value in the context of decimals
Math activity: A math game, Under 1
Exploring curriculum: analyzing mathematics activities

Class 7: Multiplying and Dividing with Decimals
Discussion of homework assignment and sharing reflections on cases
Case discussion and Math activity: multiplication of decimals; division of decimals
Discussion of Teacher Moves based on examples from the cases

Class 8: Wrapping Up
Math activity: Terminating and repeating decimals
Discussion of research: sharing participants' experiences and questions
Reviewing examples of student work from session one

Course Format:

The course meets for one 1-hour class and eight 3-hour classes. Total class hours: 25.

Reading List/Resources:

Schifter, Deborah, Virginia Bastable, and Susan Jo Russell. *Developing Mathematical Ideas: Numbers and Operations: Building a System of Tens casebook and videotapes*. White Plains, NY: Dale Seymour Publications, 1999.

MMO Syllabus

Making Meaning for Operations: In the Domain of Whole Numbers and Fractions

Course Description

Making Meaning for Operations is designed to help experienced K-8 teachers examine the actions and situations modeled by the four basic operations. It begins with a view of young children's counting strategies as they address word problems, moves to an examination of children's developing ideas of the four basic operations, and revisits the operations in the context of rational numbers. At the heart of the course are sets of classroom episodes (cases), illustrating student thinking as described by their teachers. Besides case discussions, the curriculum offers teachers opportunities to view and discuss videotapes of mathematics classrooms; to explore mathematics in lessons led by the instructor; to share and discuss the work of their own students; to plan, conduct, and analyze mathematics interviews of students; to analyze lessons taken from elementary mathematics curricula; and to read and reflect on overviews of related research.

Learning Outcomes

The goal of this course is to help participants:

1. learn more mathematics content themselves;
2. recognize key mathematical ideas with which their students are grappling;

3. appreciate the power and complexity of student thinking;
4. ask questions of students that will deepen their mathematical understanding;
5. analyze a particular activity to uncover the mathematics students will learn from it;
6. define and select mathematical objectives for their students;
7. adapt curriculum to meet individual student's needs;
8. make mathematical connections for themselves, enhancing their ability to help their students do so; and
9. connect their experiences to research.

Course Requirements

Class 1

Writing Assignment #1

Collect a work sample from three of your students involving a word problem with whole numbers or fractions. Choose one whose work you think is strong and two whose work is not so strong. For each piece of student work answer these three questions: What does the student understand? What is the student missing? What is your learning goal for each student?

Reading Assignment

Read the "Introduction" in the casebook for Making Meaning for Operations. Also read Chapter 1, including the introductory text and cases 1-7. As you read the cases, think about the questions raised in the chapter introduction.

Class 2

Writing Assignment #2 Examine the thinking of your students as they work on the two problems below. Choose numbers that are appropriate for the grade level you teach.

1. I have 375 (or 55, or 15) candy bars. I sell 90 (or 30, or 8) of them. How many candy bars do I have left?
2. I am taking a trip to visit my sister. I drive 90 (or 30, or 8) miles and then stop to rest. The total distance to my sister's house is 375 (or 55, or 15) miles. How much farther do I have to go?

Ask your students to

- make a number line model for each problem.
- write a number sentence for each problem.
- solve the number sentences.
- explain how the two problems are similar and how they are different.

Consider your students' work. What did you expect? Were you surprised? What did you learn from examining their approaches? Write up your questions, how your students responded, and what you make of their responses (your expectations, your surprises, and what you learned).

Reading Assignment #2

Read Chapter 2 of the casebook for Making Meaning for Operations, including the introductory text and cases 8-12. As you read, think about the questions raised in the chapter introduction.

Class 3

Writing Assignment #3

1. Write story problems for $5 \div 8$ so that the question you pose would be answered by each of the following.

(a) $5/8$ (b) 0.625 (c) 0 or 1 (d) 0 (e) 1

One way to approach this task is to examine the stories your group wrote for $32 \div 5$ and determine how to modify them to match $5 \div 8$.

2. For each problem, explain what it is in the story context that makes the answer appropriate for the situation.

Reading Assignment #3

Read Chapter 3 of the casebook for Making Meaning for Operations, including the introductory text and cases 13- 16. Consider the questions posed in the chapter introduction as you read.

Class 4

Writing Assignment #4

Ask your students to work on the brownie problem, 4 children sharing 7 brownies. For middle school classes, you might also pose the variation given on the planning sheet. For kindergarten and grade 1 classes, you can modify the original problem to 4 children sharing 5 brownies, or 4 children sharing 6 brownies.

Take notes on what your students did and the questions you asked to uncover their thinking. Select the work of four or five students to share at our next seminar meeting.

Write about what happened and what you learned by working with students on this task. Your writing should include the examples of student work and why each one is of interest to you.

Reading Assignment #4

Read Chapter 4 of the casebook for Making Meaning for Operations, including the introductory text and cases 17-21. Use the questions posed in the introduction as a guide for your reading of the cases.

Class 5

Writing Assignment #5A Thinking about mathematics

This assignment is about the math *you* are learning, not about the math learning of your students. Reflect on the mathematics you have been thinking about in this seminar. Choose one topic to write about. Explain how you thought about this originally, what makes sense to you now, and what aspect of the idea are you still working on. This is an opportunity to share your thinking and questions with the seminar facilitator(s), who will respond to your writing.

Writing Assignment #5B An activity from curriculum designed for students
Write answers to the questions on the “Examining Curriculum Activities” worksheet for the particular activity you are examining.

Reading Assignment #5

Read Chapter 5 of the casebook for Making Meaning for Operations, including the introductory text and cases 22-24. As you read, think about the issues raised in the chapter introduction.

Class 6

Writing Assignment #6

Pose a mathematics task to your students related to the work of this seminar. You might pose a question taken directly from one of the cases, the math activities we have been doing, or from the curriculum activity you examined in this session.

First consider why you want your students to work on this question or task, using questions 1 and 2 from the “Examining Curriculum Activities” sheet. What mathematical ideas do you want them to explore?

Then develop questions you might ask as the students are working in order to bring that mathematics to their attention (question 3). Also explore how you might modify the lesson to meet the needs of individual students (questions 4 and 5). Include your responses to these questions in your write-up.

After the session, think about what happened. What did you learn? What surprised you? Write up your questions, how your students responded, and what you make of their responses (your expectations, your surprises, and what you learned). Include specific examples of student work or dialogue. Examining the work of just a few students in depth is very helpful.

Reading Assignment

Read Chapter 6 of the casebook for Making Meaning for Operations, including the introductory text and cases 25-27. As you read, consider the questions raised in the chapter introduction.

Class 7

Writing Assignment #7

This assignment is about the math you are learning in the seminar, not about the math learning of your students. Think about the math work you have done and choose a topic to write about.

You may wish to continue thinking about the same topic you wrote about at the end of Session 4. If you choose to do that, reread both what you wrote then and the response you received from your facilitator, then explain how you are thinking about this topic now. Include examples to show what you have figured out and what you are still trying to sort out.

You may also choose to write about a different math idea. In that case, explain how you thought about this idea previously, what makes sense to you now, and what aspect of the idea are you still working on.

Reading Assignment #7

Read Chapter 7 of the casebook for Making Meaning for Operations, including the introductory text and cases 28-29. Use the questions posed in the introduction of the casebook to guide your reading

Class 8

Writing Assignment #8

After reading chapter 8, respond to the following in writing.

1. What did you learn from the research article that wasn't in the cases?
2. Identify two points in the essay that particularly interested you. Explain what made them interesting.
3. What connections did you see between your classroom and the ideas in the essay?

Reading Assignment

Read Chapter 8 of the casebook for Making Meaning for Operations, which is the essay "Highlights of Related Research."

Writing Assignment #9: Personal Reflections: Reflect and write about your experience in this course. Throughout the *Making meaning for Operations* seminar, you have been compiling a portfolio that contains your reflective writing, examples of student thinking, and cases you have written, as well as facilitator's responses to your writings. Look through the material you have collected to see how your ideas about *learning*, about *mathematics*, or about *teaching* have changed. Write a reflective paper about your changing ideas in regard to one of these three topics, referencing specific items in your portfolio as illustrations.

Evaluation

5% each	Writing Assignment 1 (total 5%)
10% each	Writing Assignments 3, 5, 7, 8, and 9 (total 50%)
15% each	Writing Assignments 2, 4 and 6 (total 45%)

Methodology

Analysis of print and video cases happens in both small group and whole class discussions. Participants work with partners to explore the mathematics in both adult and student activities presented by the instructor. Their observations and questions become the focus for class discussions. Participants also present their ideas about their students' work and about their student mathematical interviews. The instructor guides the discussion to highlight important mathematical issues and also circulates during small group and partner work to raise questions that will push participants' thinking.

Topical Outline

Class 0 (1 hour)

Introduction

Overview of course and requirements

Introduction to working with print and video cases

Explanation of initial assignment

Class 1

Making Meaning for Whole Number Addition and Subtraction

Sharing children's work samples, identifying strengths, weaknesses, and goals

Case discussion: identifying how children use counting to solve problems

Math activity: investigate the nature of subtraction

DVD: representing subtraction on a number line

Case discussion: adding and subtraction as actions

Class 2

Making Meaning for Multiplication and Division

Discussion of homework assignment and sharing reflections on cases

Discussion of learning environment: making a safe place for everyone's ideas

DVD and case discussion: examining the relationships between counting and multiplication and between the four operations

Math activity: modeling division situations

Class 3

When Dividing Doesn't Come Out Evenly

Case discussion: connecting division and fractions

Math activity: comparing fractions

Planning for student thinking assignment

Class 4

Greater Than, Less Than, or Equal to

Discussion of Student thinking assignment

Case discussion: student thinking on the relative size of fractions

Math Activity: thinking about fractional amounts

Class 5

Combining Shares, or Adding Fractions

Case discussion: examining students' strategies for adding fractions

Math activity: multiplying and dividing with fractions
DVD: seventh grade students working with 6 divided by $\frac{1}{2}$.
Examining curriculum activities

Class 6

Taking Portions of Portions, or Multiplying Fractions

Sharing participants' cases

Case discussion: identifying student strategies for multiplying fractions and reconsidering the meaning of multiplication

Math activity: considering fractions in division

Class 7

Expanding Ideas About Division in the Context of Fractions

DVD: story contexts for 4 divided by $\frac{3}{5}$

Case discussion: identifying student strategies for dividing fractions and comparing work with fractions and work with whole numbers

Math activity: operating with positive rational numbers, part one

Class 8

Wrapping Up

Discussion of research: sharing participants' experiences and questions

Math activity: operating with positive rational numbers, part two

Reviewing examples of student work from session one

Reflective writing: considering own learning in this course

Course Format

The course meets for one 1-hour class and eight 3-hour classes. Total class hours: 25.

Reading List/Resources

Schifter, Deborah, Virginia Bastable, and Susan Jo Russell. *Developing Mathematical Ideas: Numbers and Operations: Making Meaning for Operations casebook and videotapes*. White Plains, NY: Dale Seymour Publications, 1999.

EFS Syllabus

Examining Features of Shape

Course Description:

Examining Features of Shape is designed to help experienced elementary and middle-school teachers explore the features of geometric shape and how children develop an understanding of it. At the heart of the course are sets of classroom episodes (cases), illustrating student thinking as described by their teachers. Besides case discussions, the curriculum offers teachers opportunities to view and discuss videotapes of mathematics classrooms; to explore mathematics lessons led by the instructor; to share and discuss the work of their own students; to analyze lessons taken from innovative elementary mathematics curricula; and to read and reflect on overviews of related research.

Learning Outcomes:

The goal of this course is to help teachers learn how to:

- define and select mathematical objectives for their students,
- recognize key mathematical ideas with which their students are grappling,
- support their students' mathematical thinking,
- appreciate the power and complexity of student thinking,
- ask questions that deepen students' mathematical understandings,
- analyze curriculum for the mathematics it contains,
- adapt curriculum to meet the needs of individual student's needs,
- make mathematical connections for themselves, and
- connect their experiences to research.

Course Requirements:

Class 1

Writing Assignment #1: What are the key ideas related to 2D and 3D geometry that you want your students to work through over the course of the school year? Write your response before you do the casebook reading assignment. If the reading generates new ideas that you want to include in your written response, feel free to write an addendum to your comments.

Reading Assignment #1: Read the introduction to the casebook, *Examining Features of Shape*, the introduction to Chapter 1 and the six cases in Chapter One.

Class 2

Writing Assignment #2: This assignment is designed to provide an opportunity for you to examine the thinking of your students. Pose a question to your students from one of the cases from chapter one or two or from the math activities we did in the first session. Then, think about what happened: What did you expect? Were you surprised? What did you learn? Write up your question, how your students responded, and what you make of their responses. You should include examples of student work or dialogue. Examining the work of a few students, in depth, is very helpful.

Reading Assignment #2: Read Chapter 2 in the casebook, *Examining Features of Shape*, including the introduction and the five cases. As you read, consider the issues raised in the chapter introduction.

Class 3

Writing Assignment #3: Please respond to these questions:
What aspect of children's thinking has struck you? What has intrigued or surprised you?
What mathematical questions has this seminar raised for you? What mathematics are you learning?

Prepare this set of notes for the next class:

Reread the math activity worksheet and reflect on your work.

Make a list of what you have figured out.

Make a list of questions you are examining.

Reading Assignment #3 Read Chapter 3 in the casebook, *Examining Features of Shape*, including the introduction and the five cases. As you read, consider the issues raised in the chapter introduction.

Class 4

Writing Assignment #4: Pose a question about angle to your students based on our work in the seminar. You may draw from our mathematical explorations, the casebook, or the videos. Write about your question, what you expected, and how your students responded. Also, include what you expected, what, if anything, surprised you, and what you make of your students' responses. Please describe specific examples of what your students say and do. You might include actual quotations or samples of their written work. Examining the work of a few students in depth is more helpful than including the responses of every student. In your writing, be sure to include not only what your students did, but what you make of their responses. Your thinking and reflections on their work is important.

Prepare this set of notes for the next class: Reread the math activity worksheet "Angles and Angle Measures" and review your work on these questions. Summarize your current ideas about the issues raised in question #4.

Reading Assignment #4: Read Chapter 4 in the casebook, *Examining Features of Shape*, including the introduction and the five cases. As you read, consider the issues raised in the chapter introduction.

Class 5

Writing Assignment #5: Choose a mathematical idea that has been interesting or challenging for you during the seminar and write about your work on it. This assignment is about the math that you are learning in the seminar, not about the math learning of your students.

Your writing should respond to the following:

What is the math question?

How did it come up for you?

How did you work on it?

What did you figure out?

What aspects of the question are you still working on?

Reading Assignment #5: Read Chapter 5 in the casebook, *Examining Features of Shape*, including the introduction and the five cases. As you read, consider the issues raised in the chapter introduction.

Class 6

Writing Assignment #6: In this assignment, you will examine a set of curriculum materials to determine what mathematical ideas it would bring up for your students and to consider how you might highlight those ideas. Included are teacher support material and student worksheets. Read over the materials and choose **one** to focus on. You are not limited by the particulars of the lesson, such as grade level. Consider how you could use this material as a basis of work with your own students by focusing on the math ideas that can be examined.

Once you are familiar with the material, consider these questions:

What is the mathematics that I would want my students to engage with using this activity?

How would I work to bring that mathematics out?

How would I modify the lesson to make it more accessible or more challenging for my students?

What questions will I pose questions as I watch them work?

What might I learn about their understandings by listening to them or by observing them?

Reading Assignment #6: Read Chapter 6 in the casebook, *Examining Features of Shape*, including the introduction and the four cases. As you read, consider the issues raised in the chapter introduction.

Class 7

Writing Assignment #7: In this seminar you are examining a set of ideas relating to the shape of 2D and 3D geometric objects. Choose a math idea related to the seminar work and plan a lesson or a set of questions to support your own students' work on that mathematics. The lesson may be of your own design or based on the text you currently use or based on one of the curriculum activities reviewed in the seminar.

Do the activity with your class or a group of students. Write about what happened and what you learned about their thinking.

WRITE:

Before the lesson: What is the mathematics that you want your students to engage in as they do this work? What kind of questions will you ask as you observe them? What do you expect to learn from those questions and/or from your observations?

After the lesson: Describe what happened. What did your students say or do? What did you do to support their learning? What did you learn about your students' thinking? What did you learn about this mathematics? What surprised you? What pleased you? What would you do differently if you were to do this again?

Reading Assignment #7 Read Chapter 7 in the casebook, *Examining Features of Shape*, including the introduction and the four cases. As you read, consider the issues raised in the chapter introduction.

Class 8

Writing Assignment #8: Look through the entire set of cases for this seminar. Locate two cases that had an impact on your thinking---about mathematics, about the ways children learn, or about teaching. Explain in detail the way each case influenced your thinking.

Reading Assignment #8: Read Chapter 8 in the casebook, *Examining Features of Shape*. Chapter eight is not a set of teacher-written cases, but rather it contains an essay entitled “Highlights of Related Research.” In this essay, the mathematical ideas of the seminar are revisited and examined from the point of view of educational researchers who have investigated the ways children make sense of these ideas. You will see that the authors of the essay provide examples from the cases to illustrate the points examined in the research.

Personal Reflections: Your portfolio should contain your writing for each seminar assignment and facilitator responses to you. Examine this collection and then write answers to the following:

Pick one area of math you worked on in this seminar. Explain what you learned.

Pick one issue about student learning that you were thinking about during the seminar. Explain what you learned.

It is likely that there are issues, mathematical or pedagogical, that came up for you during the seminar about which you are still puzzling? Pick one issue that is still “alive” for you and explain what it is and your current thinking about it.

Evaluation:

7% Writing Assignments 1-7 (total 49%)

25% Writing Assignment 8

26% Writing Assignment 9

Methodology:

Analysis of print and video cases happen in both small group and whole class discussions. Participants work with partners to explore the mathematics in both adult and student activities presented by the instructor. Their observations and questions become the focus for class discussions. Participants also present their ideas about their students’ work. The instructor guides the discussion to highlight important mathematical issues and also circulates during small group and partner work to raise questions that will deepen participants’ thinking.

The course meets for eight three hour classes. Total class hours: 24

Reading List/Resources:

Schifter, Deborah, Virginia Bastable, and SusanJo Russell. *Developing Mathematical Ideas: Examining Features of Shape* casebook and videotapes.

MS123 Syllabus

Measuring Space in One, Two, and Three Dimensions

Developing Mathematical Ideas: Measuring Space in One, Two, and Three Dimensions

Course Description

Measuring Space in One, Two, and Three Dimensions is designed to help experienced elementary and middle-school teachers examine different aspects of size, develop facility in composing and decomposing shapes, and apply these skills to make sense of formulas for area and volume. It is

also designed to help teachers explore the conceptual issues of length, area, and volume as well as the complex interactions among these. At the heart of the course are sets of classroom episodes (cases), illustrating student thinking as described by their teachers. Besides case discussions, the curriculum offers teachers opportunities to view and discuss videotapes of mathematics classrooms; to explore mathematics in lessons lead by the instructor; to share and discuss the work of their own students; to plan, conduct, and analyze mathematics interviews of students; to analyze lessons taken from innovative elementary mathematics curricula; and to read and reflect on overviews of related research.

Learning Outcomes

The goal of this course is to help participants:

1. learn more mathematics content themselves;
2. recognize key mathematical ideas with which their students are grappling;
3. appreciate the power and complexity of student thinking;
4. ask questions of students that will deepen their mathematical understanding;
5. analyze a particular activity to uncover the mathematics students will learn from it;
6. define and select mathematical objectives for their students;
7. adapt curriculum to meet individual student's needs;
8. make mathematical connections for themselves, enhancing their ability to help their students do so; and
9. connect their experiences to research.

Course Requirements

Class 1

Writing Assignment #1

Respond in writing to the following question: What are the key ideas related to measurement that you want your students to work through during the school year? Bring three copies of your written response so it can be shared with colleagues.

Reading Assignment

Read the "Introduction" in the casebook for Measuring Space in One, Two, and Three Dimensions. Also read Chapter 1, including the introductory text and cases 1-6. As you read the cases, think about the questions raised in the chapter introduction.

Class 2

Writing Assignment #2

Pose a measurement question to your students . Then think about what happened. What did you expect? Were you surprised? What did you learn? Write up your question, how your students responded, and what you make of their responses. Include specific examples of student work or dialogue. Bring three copies of your writing to class to share with colleagues..

Reading Assignment

Read Chapter 2 of the casebook for Measuring Space in One, Two, and Three Dimensions, including the introductory text and cases 7-11. As you read, think about the questions raised in the chapter introduction.

Class 3

Writing Assignment #3

Respond to the following questions in writing:

1. How is the seminar going for you? How does it compare to what you expected?
2. In the seminar work, what aspect of children's thinking has struck you? What has intrigued you or surprised you?
3. What mathematical questions has the seminar raised for you? What mathematics are you learning? What are you working on?.

Reading Assignment

Read Chapter 3 of the casebook for Measuring Space in One, Two, and Three Dimensions, including the introductory text and cases 12-17.

Class 4

Writing Assignment #4: Thinking About Area

Please bring your mathematical thinking about the following questions to our next class:

1. Consider a rectangle whose sides are 6 inches by 1 foot. What is its area in square inches? In square feet? What does it mean to say that area = length x width?
2. Remember the crazy cakes in session 2? Here is another one: Crazy Cake E (see figure). In this trapezoid, one base is three times larger than the other. Show how you can divide the shape into two pieces to make equal shares. How can you prove the areas are equal?

Reading Assignment

Read Chapter 4 of the casebook for Measuring Space in One, Two, and Three Dimensions, including the introductory text and cases 18-21.

Class 5

Writing Assignment #5

Review the math work you did in session 4 on how to find the area of a triangle. After you have completed your math review, write your answers to these questions.

1. What mathematics has become clear to you in the process of doing and discussing the math activities in session 4?
2. How did the math work in session 4 help you interpret what is going on in Rachel's case 23? What about Sally's case 22?
3. What questions do you still have about the area of a triangle and the mathematics in this case?

4. By the next session, we will work through the mathematics in Sandra's case 24. At this point, what do you understand, and what is confusing, in Sandra's case?

Reading Assignment

Read Chapter 5 of the casebook for Measuring Space in One, Two, and Three Dimensions, including the introductory text and cases 22-24.

Class 6

Writing Assignment #6

Pose to your students a question related to volume. In your writing, include your question, how your students responded to it, and what you make of your students' responses. Please provide specific examples of what your students say and do. Please bring three copies of our writing to the next class.

Reading Assignment

Read Chapter 6 of the casebook for Measuring Space in One, Two, and Three Dimensions, including the introductory text and cases 25-28.

Math Assignment

Find some cubes and build a rectangle solid. Now build a second solid that doubles all the edges of the first. What happens to the area of each face? What happens to the surface area of the solid? What happens to the volume? What do you think would happen if you tripled the length of each edge?

Class 7

Writing Assignment #7: Examining Curriculum Materials

In this assignment, you will examine a set of curriculum materials to determine which mathematical ideas it could raise for your students and to consider how you might highlight these ideas. Read over the materials and choose one activity to focus on. Once you are familiar with the material, consider these questions:

1. When using this activity, what mathematical ideas would you want your students to work through?
2. How would you work to bring that mathematics out?
3. How would you modify the lesson to make it more accessible or more challenging to your students?
4. What questions might you ask them as you watch them work?
5. What might you learn about their understanding by listening to them or by observing them?

In our next session, you will have a chance to share ideas with others in the seminar who worked on the same curriculum activity. Please bring four copies of your writing to that session.

Math Assignment: Volume of Sand

Consider the following question about volume. Bring your work on this problem to the next session.

Sand is 6 inches deep in a 4 ft x 2 yd sandbox. What is the volume of the sand in the sandbox?

Reading Assignment

Read Chapter 7 of the casebook for Measuring Space in One, Two, and Three Dimensions, including the introductory text and cases 29-32.

Class 8

Writing Assignment #8: Reflecting on the Cases

Look back through the casebook at the entire set of cases for this seminar. Locate two cases that had an impact on your thinking—about mathematics, about the ways children learn, or about teaching. Explain in detail how each of them influenced your thinking.

Writing Assignment #8: Revisiting the Key Ideas

Before the seminar began, you identified the key ideas related to measurement that you wanted your students to work through. At our second session, you received a handout listing ideas collected from everyone in the seminar. In our next session, we will revisit this question. To prepare for this discussion, reread your first assignment and the handout summarizing the ideas collected from the class. Add comments, ideas, or modifications to your original list and bring them to the next session.

Reading Assignment

Read Chapter 8 of the casebook for Measuring Space in One, Two, and Three Dimensions, which is the essay "Highlights of Related Research." As you read, choose an issue touched on in the essay that is meaningful to you.

Evaluation

8% each	Writing Assignments 1, 3, 5, 7, 8 (total 40%)
20% each	Writing Assignments 2, 4, and 6 (total 60%)

Methodology

Analysis of print and video cases happens in both small group and whole class discussions. Participants work with partners to explore the mathematics both adult and student activities presented by the instructor. Their observations and questions become the focus for class discussions. Participants also present their ideas about their students' work and about their student mathematical interviews. The instructor guides the discussion to highlight important mathematical issues and also circulates during small group and partner work to raise questions that will push participants' thinking.

Topical Outline

Class 0 (1 hour)
Introduction

Overview of course and requirements
Introduction to working with print and video cases
Explanation of initial assignment

Class 1: Different Aspects of Size
Sharing key ideas about measurement
Case discussion
Math Activity: Ordering rectangles
Homework and exit cards

Class 2: Composing and Decomposing in One, Two, and Three Dimensions
Sharing examples of student thinking
Quick Images with video
Case discussion
Sharing crazy cake homework
Video discussion of crazy cakes
Homework and exit cards

Class 3: Measuring Length: What is a Unit and How is it Used?
Group norms discussion
Case discussion
Math activity: Changing linear dimensions
Homework and exit cards

Class 4: Measuring Area: Structuring Rectangles
Sharing homework
Case discussion
Video discussion: Crazy Cake E
Math activity: Area of Triangle
Homework and exit cards

Class 5: From Rectangles to Triangles to Trapezoids
Math discussion of homework and cases
Math activity: area formulas
Math discussion: area
Homework and exit cards

Class 6: Measuring Volume: Structuring Boxes
Math homework discussion on rectangular solids
Sharing written homework about student thinking on volume
Case discussion
Homework and exit cards

Class 7: Same Shape, Different Measures
Math homework discussion about volume of sand
Case discussion and math activity
Math discussion
Writing about the math
Sharing homework from curriculum review
Homework and exit cards

Class 8: Highlights of Related Research
Discussion of the research essay
Math discussion
Key ideas writing and discussion

Course Format

The course meets for one 1-hour class and eight 3-hour classes. Total class hours: 25.

Reading List/Resources

Schifter, Deborah, Virginia Bastable, and Susan Jo Russell. *Developing Mathematical Ideas: Measuring Space in One, Two, and Three Dimensions* casebook and videotapes. White Plains, NY: Dale Seymour Publications, 1999.

WWD Syllabus Working with Data

Course Description

Working with Data is designed to help experienced elementary and middle-school teachers examine ideas about collecting, representing, and analyzing data. At the heart of the course are sets of classroom episodes (cases), illustrating student thinking as described by their teachers. Besides case discussions, the curriculum offers teachers opportunities to view and discuss videotapes of mathematics classrooms; to explore mathematics in lessons lead by the instructor; to share and discuss the work of their own students; to plan, conduct, and analyze mathematics interviews of students; to analyze lessons taken from innovative elementary mathematics curricula; and to read and reflect on overviews of related research.

Learning Outcomes

The goal of this course is to help participants:

1. learn more mathematics content themselves;
2. recognize key mathematical ideas with which their students are grappling;
3. appreciate the power and complexity of student thinking;
4. ask questions of students that will deepen their mathematical understanding;
5. analyze a particular activity to uncover the mathematics students will learn from it;
6. define and select mathematical objectives for their students;
7. adapt curriculum to meet individual student's needs;
8. make mathematical connections for themselves, enhancing their ability to help their students do so; and
9. connect their experiences to research.

Course Requirements

Class 1

Writing Assignment #1a and 1b:

1a. In this seminar, you will be learning about ideas in data. Write the questions you have about teaching and learning about data. Your questions will likely change over the course of the seminar, but for now, what questions and thoughts are you coming in with? What are you currently thinking is important for your students to learn about data?

1b. [Participants are provided with data from School Districts A and B about the number of years the elementary teachers in each district have been teaching in grades K-6.] Your task is to represent these data in some way that helps you see what is going on. Describe in writing what you notice about the data and how the data are similar or different for the two districts.

Reading Assignment

Read the "Introduction" in the casebook for *Working with Data*. Also read Chapter 1, including the introductory text and cases 1-3. As you read the cases, think about the questions raised in the chapter introduction.

Class 2

Writing Assignment #2:

Do a short data activity with your students. Write a few paragraphs about what you did, what the students said and did, and what issues came up for you about your students' thinking. What did you expect? Were you surprised? What did you learn? Write up your question, how your students responded, and what you make of their responses. Include specific examples of student work or dialogue. Bring three copies of your writing to class to share with colleagues.

Data Project: For the next session, think about ideas for a question that could lead to a data investigation you would like to do with a partner or small group of seminar participants. The questions might relate to some issue about education, about your school or community, or just something that interests you. Come next time with at least two questions you might want to pursue. Keep in mind that the investigation should involve numerical data. Your project should also involve comparing at least two groups.

Reading Assignment

Read Chapter 2 of the casebook for *Working with Data*, including the introductory text and cases 4-6. As you read, think about the questions raised in the chapter introduction.

Class 3

Writing Assignment #3

Respond to the following questions in writing:

1. How is the seminar going for you? How does it compare to what you expected?
2. In the seminar work, what aspect of children's thinking has struck you? What has intrigued you or surprised you?

3. What mathematical questions has the seminar raised for you? What mathematics are you learning? What are you working on?

Collect data about your morning commute. How long does it typically take you to get from home to school in the morning? Before our next class, collect data for each day you travel from home to school. Looking at your data, choose a number that you think best represents the amount of time it typically takes for you to get from home to school.

Data Project: Form a group of two to five people for your data investigation. What is your question? What data do you need? How will you collect them? How will your data provide information about your question? Plan a pilot investigation to try out your question and data collection methods.

Reading Assignment

Read Chapter 3 of the casebook for Working with Data, including the introductory text and cases 7-11.

Class 4

Writing Assignment #4:

Do a pilot test for your data investigation. With your small group, decide how you will test the design of your investigation. For the next session, bring a representation of the data you collected and a few paragraphs that include a description of your data, what you learned from doing the pilot, what questions you now have, and your thoughts about revising your data collection methods.

Make a representation of the group's data on commute time to school. Find the median. Write a few sentences describing the data as if you were reporting it to someone who did not have the actual data in front of them.

Reading Assignment

Read Chapter 4 of the casebook for Working with Data, including the introductory text and cases 12-16.

Class 5

Writing Assignment #5

In this assignment you will examine a set of curriculum materials to determine which mathematical ideas it could raise for your students and to consider how you might highlight those ideas. Read over the materials and choose one activity to focus on and consider these questions:

1. When using the activity, what mathematical ideas would you want your students to work through?
2. How would you bring that mathematics out?
3. How would you modify the lesson to make it more accessible or more challenging for your students?
4. What questions might you ask them as you watch them work?

5. What might you learn about their understanding by listening to them or by observing them?

In our next session, you will have a chance to share ideas with others in the seminar who worked on the same curriculum activity. Please bring four copies of your writing to that session.

Finish working on the math activity we began in class and be ready to discuss your work about the next session.

Reading Assignment

Read Chapter 5 of the casebook for Working with Data, including the introductory text and cases 17-19.

Class 6

Writing Assignment #6

Write about a data experience with your students in which they compare two groups. Recount what the students said and did using as much detail as you can. As you consider your students' thinking, write about what stood out for you, what confused you, what surprised you, what your questions are. As part of your writing, consider this question: How are your students making sense of the "big picture" of the data?

Reading Assignment

Read Chapter 6 of the casebook for Working with Data, including the introductory text and cases 20-24.

Class 7

Writing Assignment #7:

Look back through the casebook at the cases you have read so far for this seminar. Choose two cases that have helped you think about issue of learning and teaching in your own practice. Write about how these cases affected you thinking or raised questions for you about the important ideas in collecting, representing, and analyzing data. How do you think these cases may affect your work with your own students? This writing can be primarily about the questions you are raising for yourself rather than about any conclusions you have come to.

Do the problems on Averages Part 1. Continue working on your data project.

Reading Assignment

Read Chapter 8, the Highlights of Related Research. (You will be reading Chapter 7 for the final session.)

Class 8

Writing Assignment #8:

With your group, get ready to set up your station to present your data project. Consider what you will need in order to make clear what you did and what you found out. You will have time to circulate and look at other group's presentations. Then you will have a short time to summarize your findings to the group, to explain the issues you faced, and to hear questions and comments from others. You will also need to hand in a written report of your project including any graphs or representations of your data.

Reading Assignment

Read Chapter 7 of the casebook for Working with Data, including the introductory text and cases 25-28.

Evaluation

8% each	Writing Assignments 1, 3, 5, 7, 8 (total 40%)
20% each	Writing Assignments 2, 4, and 6 (total 60%)

Methodology

Analysis of print and video cases happens in both small group and whole class discussions. Participants work with partners to explore the mathematics both adult and student activities presented by the instructor. Their observations and questions become the focus for class discussions. Participants also present their ideas about their students' work and about their student mathematical interviews. The instructor guides the discussion to highlight important mathematical issues and also circulates during small group and partner work to raise questions that will push participants' thinking.

Topical Outline

Class 0 (1 hour)

Introduction

Overview of course and requirements

Introduction to working with print and video cases

Explanation of initial assignment

Class 1: Getting Started with Data

Data activity

Case discussion

Video of Pocket Data

Homework and exit cards

Class 2: Designing a Data Presentation

Sharing examples of student thinking

Case discussion

Data Activity: Surveys of our group

Homework and exit cards

Class 3: Categorical and Numerical Data

Group norms discussion

Case discussion

Math activity: Describing Numerical Data

Homework and exit cards

Class 4: Numerical Data: What Do the Numbers Mean?

Sharing pilot data

Video and case discussion

Math activity: Median as a Tool

Homework and exit cards

Class 5: Comparing Data Sets

Math activity: Lost Teeth/Stem and Leaf Plots

Video case

Case discussion

Homework and exit cards

Class 6: Average: Developing Ideas About Middle

Sharing written homework

Case discussion

Math activity: Box Plots

Homework and exit cards

Class 7: The Mean/Research Highlights

Math activity: Working with the Mean

Research Essay

Homework and exit cards

Class 8: Using Averages/Project Presentations

Video and case discussion

Project presentations

Reflections on the course

Course Format

The course meets for one 1-hour class and eight 3-hour classes. Total class hours: 25.

Reading List/Resources

Schifter, Deborah, Virginia Bastable, and Susan Jo Russell. Developing Mathematical Ideas: Working with Data casebook and videotapes.

RAO Syllabus

Reasoning Algebraically About Operations: In the Domains of Whole Numbers and Integers

Course Description:

Reasoning Algebraically about Operations is one of two DMI modules on early algebraic thinking. In this module, participants examine generalizations at the heart of the study of operations in the elementary grades. They express these generalizations in common language and in

algebraic notation, develop arguments based on representations of the operations, study what it means to prove a generalization, and extend their generalizations and arguments when the domain under consideration expands from whole numbers to integers.

The DMI seminar, *Making Meaning for Operations*, is a prerequisite to *Reasoning Algebraically about Operations*.

Learning Outcomes:

The goals of this course are to help teachers learn how to:

- Articulate mathematical generalizations in common language and in algebraic notation.
- Develop arguments for mathematical generalizations through the use of physical and visual representations that can accommodate a class of numbers.
- Use contexts and representations to make sense of addition and subtraction of integers and rules for ordering integers.
- Become familiar with the laws of arithmetic as the basis of mathematical proof in algebra.
- Participate in conversations with colleagues about cases of students making and justifying generalizations in the context of their work on numbers and operations.
- Write cases about their own students' engagement with the ideas of the seminar.

Course Requirements:

In preparation for each of the eight sessions, participants complete a written assignment and read a chapter of cases or, in the eighth session, an essay about the ideas of the seminar .

Class 1

Writing assignment: Examples of student thinking

It is likely that reading the cases and working on the math in this seminar have made you curious about how your own students might think about odd and even numbers. This assignment asks you to examine the thinking of your students.

Pose a question to your students related to odd and even numbers. You might pose a question taken directly from one of the cases in chapter 1. Then think about what happened. What did you expect? Were you surprised? What did you learn? Write up your question, how your students responded, and what you make of their responses (your expectations, your surprises, and what you learned). Include specific examples of student work or dialogue. Examining the work of just a few students in depth is very helpful.

At our next session, you will have the opportunity to share this writing with other colleagues. Please bring three copies of your writing to turn in.

Note: You will be asked to prepare similar assignments that involve investigating students' thinking for Session 4 and Session 6. Check your classroom schedules and lesson plans to be sure you will be able to complete these assignments.

Class 2

Writing assignment: Questions about mathematics

What questions about mathematics have the first two sessions brought up for you? In about one or two pages, explain the mathematical questions you are wondering about as a result of this seminar.

Class 3

Writing assignment:

Pose a question to your students related to the work of this seminar to examine how they think about these issues. Think strategically about how to create situations in which children will have a reason to make general statements about arithmetic and create models to show their thinking.

Then think about what happened. What did you expect? Were you surprised? What did you learn? Write up your question, how your students responded, and what you made of their responses (your expectations, your surprises, and what you learned). Include specific examples of student work or dialogue. Examining the work of just a few students, in depth, is very helpful.

At our next session, you will have the chance to share this writing with colleagues. Please bring three copies of your writing to turn in.

Class 4

Writing assignment: Pursuing a mathematical question

This assignment is about the math *you* are learning in the seminar, not about the math learning of your students. Reflect on the mathematics you worked on in this session by considering two questions: What makes sense to you now; what are you still working on?

Take some time between this session and the next to reflect on the math ideas from this session. What are those ideas? What of those ideas make sense to you now? What of those ideas are you still working on? How are you working on them?

Class 5

Math assignment before reading Chapter 6.

Here is a problem we will be reading about for next session. Solve this problem using a variety of approaches:

On the weekend I found many flowers in my garden. In the morning I picked 4 bunches of flowers to give to my family. That afternoon I picked 3 more bunches to give to some friends. Each bunch had 8 flowers. How many flowers did I pick?

Writing assignment: Examples of student thinking

Pose a question to your students related to the ideas in chapter 4, 5, or 6 of the casebook. Then think about what happened. What did you expect? Were you surprised? What did you learn? Write up your question, how your students responded, and what you make of their responses. Include specific examples of student work or dialogue. Examining the work of just a few students, in depth, is very helpful.

At our next session, you will have the chance to share this writing with colleagues. Please bring three copies of your writing to turn in.

Class 6

Writing assignment 1:

Previous seminar assignments have involved you in examining the mathematical thinking of your students or in engaging with mathematics for yourself. At this point in the seminar, we are interested in how your ideas about teaching and learning mathematics have been influenced by the seminar experience. Please write about this.

Writing assignment 2:

Look over the page, “The Laws of Arithmetic.” As I have stated in that document, I am interested in knowing your reaction to these formal statements and how they connect with the work we have done together in the seminar.

Class 7

Writing assignment: Reflecting on Chapter Eight

Pick three points in the essay that particularly interested you. Write about each of those points: Explain what made it interesting. How do you connect to those ideas?

Pick two points in the essay that you found confusing or with which you disagree. Explain each of these. What was confusing or how is your own experience different from what is suggested?

Bring this writing to the next session to support discussion of this chapter.

Class 8

Final Reflection Questions

This is an opportunity for you to think through your experiences in the seminar. Read through the collection of your assignments, facilitator's responses, your exit cards, and your math work to get a sense of how your ideas have changed. Use this body of work to guide your responses to the following questions.

1. How have your ideas about algebra changed over the course of the seminar? Be specific about the ideas that have changed.
2. How have your ideas about learning changed? Be specific.
3. How have your ideas about teaching changed? Be specific.
4. Likely there are issues, mathematical and pedagogical, that came up for you during the seminar that still puzzle you. Pick one issue that is still "alive" for you. Explain what it is and your current thinking about it.

Evaluation:

50%	Participation in small group and whole group discussions
50%	Written Homework Assignments

Methodology:

At the heart of the materials are sets of classroom episodes (cases) illustrating student thinking as described by their teachers. In addition to case discussions, the curriculum offers teachers opportunities: to explore mathematics in lessons led by facilitators; to share and discuss the work of their own students; to view and discuss videotapes of mathematics classrooms; to write their own classroom episodes;; and to read an essay that addresses the ideas explored in the seminar from a mathematician's perspective.

Topical Outline:

Session 1 (3 hours): Reasoning in the Context of Even and Odd Numbers

Mathematical themes:

- Are two different definitions of even numbers equivalent?
- How do odd and even numbers fare under the operations of addition and multiplication?
- How do you prove a statement is always true when you can't check all the numbers?

Sharing homework Introductions	Groups of 3 Whole Group	20 minutes
Definitions of even Definitions of even discussion	Groups of 3 Whole group	25 minutes
Video Session 1 clip 1	Whole group	15 minutes
Math: Adding odd numbers	Whole group	20 minutes
Video Session 1 clip 2	Whole group	10 minutes
Break		15 minutes
Math activity: Multiplying odd numbers	Small groups Whole group	20 minutes 20 minutes
Small-group case discussion Whole-group case discussion	Groups of 3 Whole group	30 minutes
Homework and exit cards	Whole group	5 minutes

Session 2 (3 hours): Finding relationships in addition and subtraction

Mathematical themes:

- What are the generalizations that underlie various computational strategies for addition and subtraction?
- How can we justify these generalizations?
- How do we draw on models for representing addition and subtraction, such as visual images, story contexts, and number lines, to express and justify generalizations?

Sharing student thinking assignment	Groups of 3	35 minutes
Generalizations in Lola's case	Whole group	20 minutes
Video Session 2	Whole group	20 minutes
Math discussion	Whole group	30 minutes
Break		15 minutes
Case discussion	Groups of three Whole group	55 minutes
Homework and exit cards	Whole group	5 minutes

Session 3 (3 hours): Reordering Terms and Factors

Mathematical Themes:

- How can we justify that switching the order of addends results in the same sum and that switching factors results in the same product? (That is, how can we demonstrate the commutative property for addition and multiplication?)
- What patterns occur when switching the order of the numbers in a subtraction or a division problem and how can we explain these patterns?
- What similarities between the structure of addition and multiplication does this work suggest?

Norms Discussion	Whole Group	15 minutes
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Video & Case discussion (FQ 1-6)	Groups of 3	40 minutes
Math discussion (FQ 6)	Whole group	30 minutes
BREAK		15 minutes
Case discussion (FQ (7 &/or 8)	Small group	30 minutes
Math discussion (FQ 7 &/or 8)	Whole group	30 minutes
Planning work with students	Groups of 3	15 minutes
Exit cards	Whole group	5 minutes

Session 4 (3 hours): Expanding the Number System

Mathematical themes:

- What are situations in which negative numbers are useful?
- What are the limitations and strengths of two models (number line and charge) for operating with negative numbers?
- As the number system is extended to include 0 and negative numbers, how do we need to modify our ideas of what it means to order and to add numbers?

Sharing homework (episodes)	Groups of 3	30 minutes
Math activity	Whole group/Groups of 3	40 minutes
Math discussion	Whole group	30 minutes
BREAK		15 minutes
Case discussion	Small group	30 minutes
Video and Case discussion	Whole group	30 minutes
Homework and exit cards	Whole group	5 minutes

Session 5 (3 hours): Doing and undoing, staying the same

Mathematical themes:

- What are ways to model and express the inverse relationship between addition and subtraction; i.e. if $a + b = c$, then $c = b - a$?
- How are the roles of 0 (in addition) and of 1 (in multiplication) the same? How are they different?
- How can we use the inverse relationship between addition and subtraction to make sense of subtraction with negative numbers?

Examining Addition and Subtraction	Groups of 3	30 minutes
Video and Discussion	Whole group	35 minutes
Break		15 minutes
Cases 23& 24	Small groups	20 minutes
Case discussion (role of 1 and 0)	Whole group	25 minutes
Math activity (examining subtraction with negative numbers)	Small groups	30 minutes
Math discussion	Whole group	20 minutes
Exit cards	Whole group	5 minutes

Session 6 (3 hours): Multiplying in clumps

Mathematical themes:

- How is the distributive property called upon in computational strategies for multiplication and division?
- What are common misapplications of the distributive property and how can we use these errors to deepen understanding of multiplication and division?
- What does it mean to “understand” the distributive property?

Sharing homework (episodes)	Groups of 3	30 minutes
Math activity	Groups of 3	35 minutes
	Whole group	35 minutes
Video	Whole group	10 minutes
BREAK		15 minutes
Case discussion	Small group	25 minutes
	Whole group	25 minutes
Homework and exit cards	Whole group	5 minutes

Session 7 (3 hours): Exploring rules about factors

Mathematical themes:

- What are the relationships among the factors of a number
- What are the relationships among a number and its factors?
- How can story contexts, diagrams, and cube structures be used to model relationships among factors?
- How do we draw on such models to express and justify generalizations?

Reaction to “Laws of Arithmetic”	Whole group	30 minutes
Math and Case Discussion	Groups of 3	40 minutes
	Whole group	30 minutes
BREAK		15 minutes
Examining Teacher Moves	Small group	35 minutes
	Whole group	25 minutes
Homework and exit cards	Whole group	5 minutes

Session 8 (3 hours): Wrapping up

Mathematical themes:

- What does it mean to state and justify a generalization? What does mathematical justification look like in an elementary classroom?
- What are the connections between various representations (such as diagrams or drawings, cube structures, story contexts) and algebraic notation?

Chapter 8 Discussion	Small groups	35 minutes
Chapter 8 Discussion	Whole group	25 minutes
Break	Break	15 minutes
Math Activity	Small groups	40 minutes
Math Discussion	Whole group	35 minutes
Closing	Whole group	30 minutes

Course Format:

The course meets for eight 3-hour sessions. Total: 24 hours.

Reading List/Resources:

Schifter, Deborah, Virginia Bastable, and Susan Jo Russell. Reasoning Algebraically about Operations casebook and videotapes.

PFC Syllabus Patterns, Functions, and Change

Patterns, Functions, and Change

Course Description

Patterns, Functions, and Change is designed to help experienced elementary and middle-school teachers examine the connections between repeating patterns and mathematical functions which describe how two quantities change in relationship to one another. It is also designed to help teachers explore the conceptual issues children and adults face as they work to represent the relationships between two quantities using tables, graphs, arithmetic rules and symbolic notation. The seminar includes a study of a variety of functions (linear, quadratic, and exponential), the graphs for these functions, and how these representations show how two quantities change in relation to one another. In addition, graphs of functions that are not described by rules but are determined by empirical data such as temperature over time or growth over time, are also studied. At the heart of the course are sets of classroom episodes (cases), illustrating student thinking as described by their teachers. Besides case discussions, the curriculum offers teachers opportunities to view and discuss videotapes of mathematics classrooms; to explore mathematics in lessons lead by the instructor; to share and discuss the work of their own students; to plan, conduct, and analyze mathematics interviews of students; to analyze lessons taken from innovative elementary mathematics curricula; and to read and reflect on overviews of related research.

Learning Outcomes

The goal of this course is to help participants:

1. learn more mathematics content themselves;
2. recognize key mathematical ideas with which their students are grappling;
3. appreciate the power and complexity of student thinking;
4. ask questions of students that will deepen their mathematical understanding;
5. analyze a particular activity to uncover the mathematics students will learn from it;
6. define and select mathematical objectives for their students;
7. adapt curriculum to meet individual student's needs;
8. make mathematical connections for themselves, enhancing their ability to help their students do so; and
9. connect their experiences to research.

Course Requirements

Class 1

Reading Assignment

Read the "Introduction" and Chapter One (cases 1 - 4) of the Casebook, *Patterns, Functions and Change*. As you read the cases, think about the questions raised in the chapter introduction.

Writing Assignment #1

Chapter One includes three math activities embedded in the case reading. Please write out your work on these problems and bring them to the first session. At the end of the session facilitators will collect this work.

Class 2

Writing Assignment #2

Pose a math task to your students related to number sequences. You might pose a question taken directly from one of the cases. Then think about what happened. What did you expect? Were you surprised? What did you learn? Write up your question, how your students responded, and what you make of their responses (your expectations, your surprises, and what you learned). Include specific examples of student work or dialogue. Examining the work of just a few students in depth is very helpful. Bring three copies of your writing to class to share with colleagues.

Reading Assignment

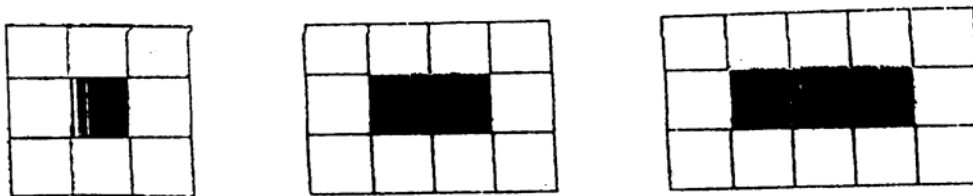
Read Chapter 2 of the casebook for *Patterns, Functions and Change* including the introductory text and cases 5 - 8. As you read, think about the questions raised in the chapter introduction.

Class 3

Writing Assignment #3

BEFORE you read the cases, spend some time working on the mathematics involved for yourself. Bring your work to the next session so you can use it in discussion with other participants. You will also hand in your work on these problems to the facilitator.

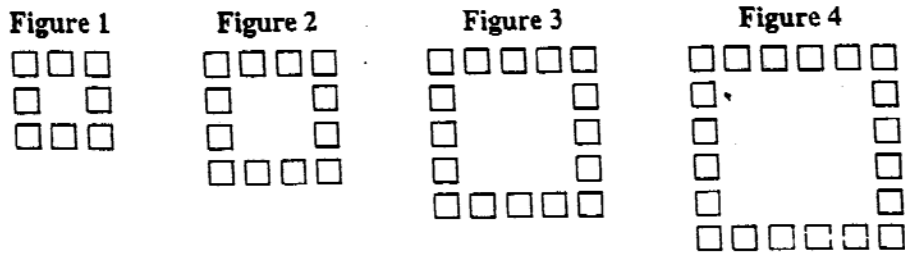
1. These figures are made up of white and dark tiles. For 1 dark tile, 8 white tiles are needed, for 2 dark tiles, 10 dark tiles are needed and so on. How many white tiles for 10 dark tiles? For 100 dark tiles? For n dark tiles?



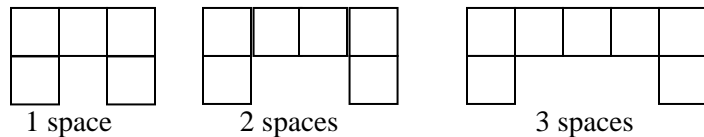
2. In the chicken coop the hens roost in a line. Each hen has her own roost. Henrietta's roost is in the

middle. If Henrietta is in the 7th position, how many hens are in the line? If Henrietta is in the 10th position, how many hens are in the line? If Henrietta is in the nth position, how many hens are in the line?

3. How many square tiles are needed to build figure 10? Write an expression for the 100th case? Can you write a rule for the nth case? Explain your thinking.



4. In the pattern below: 5 blocks are needed to form 1 space, 6 blocks are needed to form 2 spaces, and 7 blocks are needed to form 3 spaces. How many blocks for 10 spaces? 100 spaces? N spaces?



Reading Assignment

Read Chapter 3 of the casebook for Patterns, Functions, and Change including the introductory text and cases 9 - 12.

Class 4

Writing Assignment #4:

Pose a math task to your students related to the work of this seminar. You might pose a question taken directly from one of the cases or the math work. First consider why you want your students to work on this question. What mathematical ideas do you want them to explore? Then consider what questions you might ask as they are working in order to bring that mathematics to the fore. You can use the planning worksheet from the small group discussion as a support.

After the session, think about what happened. What did you learn? What surprised you? Write up your question, how your students responded, and what you make of their responses (your expectations, your surprises, and what you learned). Include specific examples of student work or dialogue. Examining the work of just a few students in depth is very helpful.

At our next session, you will have the opportunity to share this writing with the colleagues from your small group planning session. Please bring four copies of your writing to share.

Reading Assignment

Read Chapter 4 of the casebook for Patterns, Functions, and Change, including the introductory text and cases 13 - 15.

Class 5

Writing Assignment #5

Carefully examine the lesson from the NCTM publication series, *Navigating through Algebra* for the grade level you teach. Try the math yourself. What math ideas can the lesson highlight? **DO NOT DO THE LESSON WITH STUDENTS YET.** You will have an opportunity to work on this lesson with your own students and write about it for session 6.

For session 5, write about how you will use this material to bring out the ideas you are interested in exploring with your students by responding to the following planning questions:

- 1) What mathematical ideas do you want your students to work on in doing this lesson?
- 2) What do they already know that they might use to do this work?
- 3) What range of strategies or approaches do you expect to see in your class?
- 4) What questions might you ask as students are working to bring out the math ideas?
- 5) What will be the main focus of the whole group discussion?

Bring 3 copies of this planning work and the curriculum materials to session 5 so you will be able to discuss your ideas with others.

Reading Assignment

Read Chapter 5 of the casebook for Patterns, Functions, and Change, including the introductory text and cases 16 to 21

Class 6

Writing Assignment #6

Pose the math task you have chosen and modified from the NCTM resource. Write about what happened:

- What did you learn?
- How did your students respond?
- What do you make of their responses?
- What surprised you?

Include specific examples of student work or dialogue. Examining the work of just a few students in depth is very helpful.

Reading Assignment

Read chapter 6 in the casebook, Examining Non-Constant Change, including the introductory text and cases

22 – 25.

Class 7

Writing Assignment #7:

Previous seminar assignments have involved you in examining the mathematical thinking of your students or in engaging with mathematics for yourself. At this point in the seminar, we are interested in how your ideas about teaching and learning mathematics have been influenced by the seminar experience. Please write about this.

Reading Assignment

Read chapter 7 in the casebook, Functions without Rules including the introductory text and cases 26 to 28.

Class 8

Writing Assignment #8:

Locate two or three points in the essay that particularly interested you. Write about each of those points: Explain what made it interesting. What experiences have you had that led you to connect to these ideas?

Locate two or three points in the essay that you found confusing or which seem not to describe experiences you have had. Explain each of these. What was confusing or how was your own experience different from what is suggested?

Locate one or two points in the essay that suggest something you want to incorporate into your teaching practice. Explain each of these.

Bring 2 copies of this writing to the next session to support discussion of this chapter.

Reading Assignment

Read Chapter 8, the essay, “An Introduction to the Mathematics of Patterns, Functions, and Change” by Steve Monk.

Evaluation

8% each	Writing Assignments 1, 3, 5, 7, 8 (total 40%)
20% each	Writing Assignments 2, 4, and 6 (total 60%)

Methodology

Analysis of print and video cases happens in both small group and whole class discussions. Participants work with partners to explore the mathematics both adult and student activities presented by the instructor. Their observations and questions become the focus for class discussions. Participants also present their ideas about their students' work and about their student mathematical interviews. The instructor guides the

discussion to highlight important mathematical issues and also circulates during small group and partner work to raise questions that will push participants' thinking.

Topical Outline

Class 0 (1 hour)

Introduction

Overview of course and requirements

Introduction to working with print and video cases

Explanation of initial assignment

Class 1: Using Patterns to Predict

Group Norms

Math Activity: Cube Trains

Video

Case discussion

Homework and exit cards

Class 2: Representing Situations with Diagrams, Tables, and Rules

Sharing examples of student thinking

Math activity: Comparing Functions

Case discussion

Math Homework

Exit cards

Class 3: Finding Rules

Case discussion

Video

Math activity: Comparing Penny Jars

Preparing for student thinking assignment

Homework and exit cards

Class 4: Comparing Functions

Sharing homework

Math activity: Catching Up or Not?

Video

Case discussion

Homework and exit cards

Class 5: Does Doubling Work?

Case discussion

Video

Math activity: Growing Patterns

Planning for student thinking assignment

Homework and exit cards

Class 6: Examining Non-Constant Change

Sharing examples of student thinking

Math Discussion – Growing patterns

Case discussion

Examining Teacher Moves

Homework and exit cards

Class 7: Functions Without Rules

Math activity: Stories and Graphs

Video

Case discussion

Homework and exit cards

Class 8: Understanding the mathematics of Patterns, Functions, and Change for the K-8 Classroom

Discussion of the essay

Math activity: Growing and Painted Cubes

Portfolio Review and close

Course Format

The course meets for one 1-hour class and eight 3-hour classes. Total class hours: 25.

Reading List/Resources

Schifter, Deborah, Virginia Bastable, and Susan Jo Russell. *Developing Mathematical Ideas: Patterns Functions, and Change* casebook and videotapes.